Mech
Sem-III

Seat No. : ____________

7695

Applied Mathematics – II

Time : 3 Hours] **[Max. Marks : 100**
(2 : 30 P.M. to 5 : 30 P.M.)

Instructions :
(1) Attempt **all** questions.
(2) Answer to the two sections must be written in separate answer.
(3) Figures to the right indicates full marks.
(4) Assume suitable data, if necessary.

SECTION – I

1. Attempt any **three** : **18**

(a) Express $f(x) = \frac{1}{4}(\pi - x)^2$ as a Fourier Series with period 2π to be valid in the interval 0 to 2π.

(b) Find the Fourier Series expansion for periodic function $f(x)$, if

$$f(x) = -\pi \quad , -\pi < x < 0$$
$$= x \quad , \quad 0 < x < \pi.$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$

(c) Find the Fourier Series to represent $f(x) = x^2 - 2$, when $-2 \le x \le 2$. $P = 2l = 4$, $l = 2$.

(d) Express $f(x) = c - x$ when $0 < x < c$ as a half-range cosine series with period 2c.

(e) Find the half-range sine series to represent $f(x) = lx - x^2$ in the range $(0, l)$

2. (a) Attempt any **two** : **6**

Find : (i) $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$ (ii) $L\{t^2 \cdot \sin 4t\}$

(iii) $L\left\{\int_0^\infty e^{-tx^2}\, dx\right\}$

(b) Evaluate any **two** : **6**

(i) $L^{-1}\left\{\frac{S}{S^4+4a^2}\right\}$ (ii) $L^{-1}\left\{\frac{1}{S^3(S^2+a^2)}\right\}$

(iii) $L^{-1}\left\{\frac{e^{-25}}{S^2+4}\right\}$

(c) Solve the equation **4**

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\cdot \sin t\,,\ x(0)\,,\ x'(0) = 1.$$

3. (a) Find the Laplace transform of the function **4**

$$f(t) = \begin{cases} \sin wt & ; \ 0 < t < \frac{\pi}{w} \\ 0 & ; \ \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases} \qquad f\left(t + \frac{2\pi}{w}\right) = f(t).$$

OR

If $L\{f(t)\} = \bar{f}(s)$ then prove that $L\{f(t-a)\cdot u(t-a)\} = e^{-as}\cdot\bar{f}(s)$. Hence find the Laplace transform of $u(t-\pi)\cdot\cos t$.

(b) Attempt any **three** of the following : **12**

(i) $(D^2 + 5D + 4)\,y = x^2 + 7x + 9.$

(ii) $(D^2 + 9)\,y = x\sin 2x.$

(iii) Solve by the method of variation of parameter $(D^2 + 4)\cdot y = \tan 2x.$

(iv) A body executes damped forced vibration given by the equation

$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + b^2x = e^{-kt}\sin wt.$ Solve the equation when

(i) $w^2 \neq b^2 - k^2$

(ii) $w^2 = b^2 - k^2.$

OR

(iv) Solve the following simultaneous equations

$$\frac{dx}{dt} + 2\frac{dy}{dt} - 2x + 2y = 3e^t$$

$$3\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 4e^{2t}.$$

SECTION – II

4\. (a) Form the Partial differential equation (any **one**) :

(i) $2z = (ax + y)^2 + b$, where b is constant.

(ii) $z = f(x + ct) + g(x - ct)$.

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(b) Attempt any **two** :

(i) Solve : $x^2 (y - 3)p + y^2 (3 - x)q = z^2 (x - y)$.

(ii) Solve : $(mz - ny)p + (nx - lz)q = ly - mx$.

(iii) Solve : $p + q = \sin x + \sin y$.

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(c) Solve the equation $\frac{\partial 2z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by separation of variable.

OR

(c) A tightly stretched string with fixed ends $x = a$ & $x = l$ is initially at rest is in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l - x)$, find its displacement.

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5\. Attempt any **four** :

(a) Show that the all circles $x^2 + y^2 = 4x$ transforms into straight line $4u + 3 = 0$ under the transformation $w = \frac{2z + 3}{z - 4}$.

(b) Find the bilinear transformation which maps points i, i, –1 onto i, 0, – i also find its invariant points.

(c) Show that the image of hyperbola $x^2 - y^2 = 1$ is $R^2 = W_{12\phi}$ under the mapping $w = 1/z$.

(d) Define Harmonic function, show that the function $u = x^2 - y^2 - y$ is harmonic & construct the corresponding analytic function.

(e) If f(z) is an analytic function then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4\,|f'(z)|^2$.

(f) Define : Analytic function, conformal transformation, Orthogonal system. State necessary & sufficient condition for the function to be an analytic.

P.T.O.

6. (a) Attempt any **two** : **8**

(i) Prove that $J_n'(x) + \frac{n}{x} J_n(x) = J_{n-1}(x)$.

(ii) Show that $x^4 = \frac{1}{35}\,[8\,p_4(x) + 20\,p_2(x) + 7\,p_0(x)]$.

(iii) Solve : $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$.

(b) Solve in series the equation $\frac{d^2y}{dx^2} + x^2 y = 0$ **4**

OR

(b) Solve the equation : $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2 y = 0$. Where n is constant. **4**

(c) Solve any **two** : **6**

(i) $\displaystyle\int_{c\,:\,|z|=1} \frac{\sin^3 z}{\left(z - \frac{\pi}{6}\right)^3}\, dz$

(ii) $\displaystyle\int_{c\,:\,|z|=3/2} \frac{\cos \pi z^2}{(z-1)(z-2)}\, dz$

(iii) $\displaystyle\int_{c\,:\,|z|=3} \frac{e^{2z}}{(z-1)(z-2)}\, dz$
